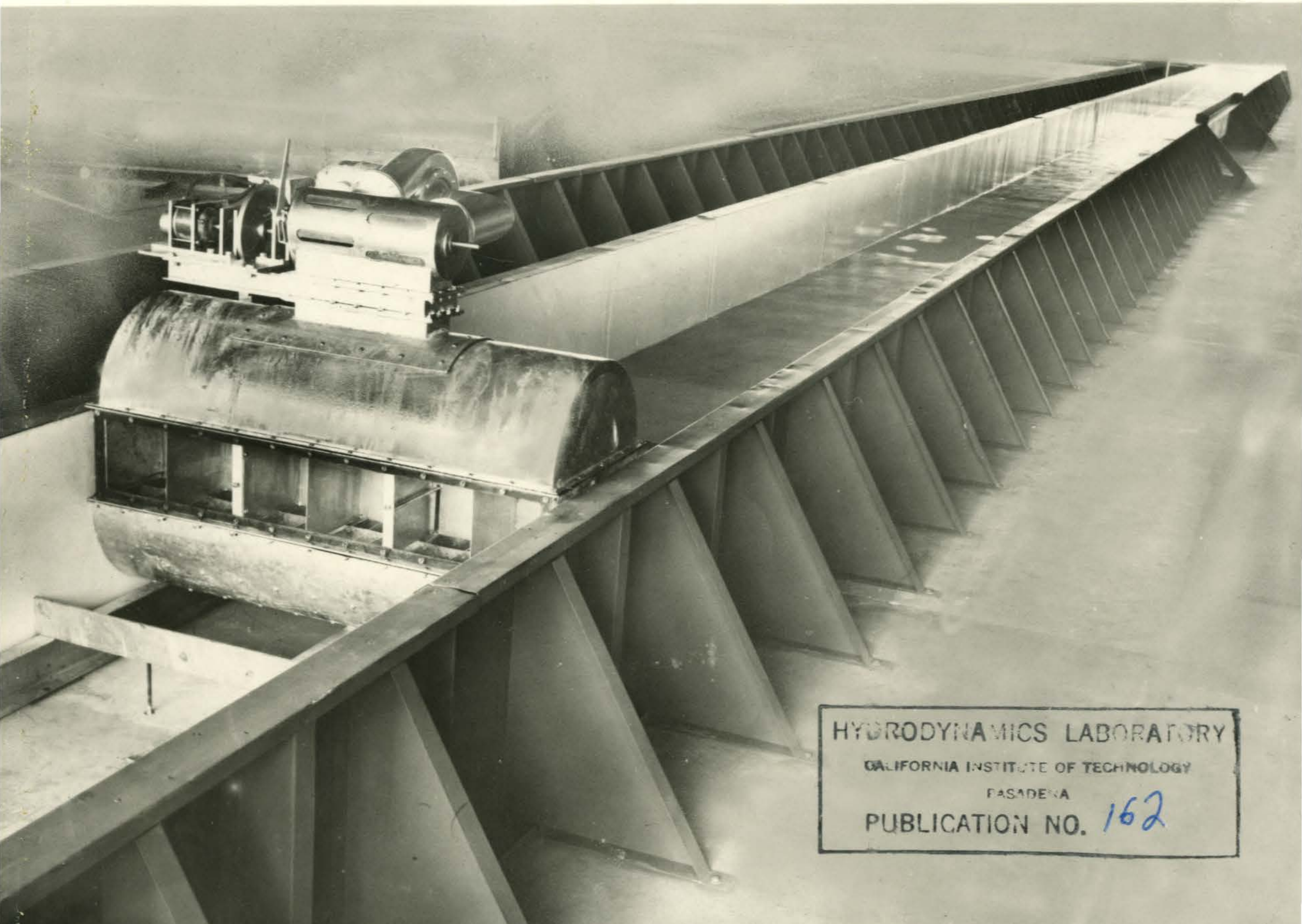
MODEL STUDIES
OF
MOBILE BREAKWATERS

PROGRESS REPORT for DECEMBER, 1949



CALIFORNIA INSTITUTE OF TECHNOLOGY
Hydrodynamics Laboratories, Hydraulic Structures Division

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Progress Report for December 1949

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MOBILE BREAKWATERS

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The Cover . . .

The cover photograph shows the 4-foot wide by 130-foot long wave channel at the Azusa Laboratory. The wave machine used with this channel is capable of generating waves up to 4 inches in height and from 2 to 30 feet in length. The transparent wall section is useful for observing the particle motion associated with the wave trains and for obtaining photographs of wave profiles.

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Model Studies of Mobile Breakwaters

I. INTRODUCTION

Two general physical processes by which wave energy may be excluded from a region are wave reflection and wave interference at the seaward boundary of the region. Wave reflection is the common basis of operation of conventional breakwaters, and for such structures is accompanied by the development of large forces and overturning moments. Wave interference is also a common occurrence although less generally understood, being responsible for the characteristic diffraction effects observed when waves pass through a breakwater gate.

Because of the large forces developed, a mobile breakwater designed to totally reflect incident storm waves appears to be an impossibility, but there is a good possibility that submerged breakwaters, which are partially reflecting barriers, may be designed which will provide sufficient protection to be useful and subjected to forces small enough to permit installation and maintenance. In addition, if several such partially reflective barriers are installed in series, it may be possible to take advantage of wave interference to increase greatly their net effect.

The Laboratory is presently engaged in a study of this system of mobile breakwaters. The factors to be investigated include:

(1) Determination of reflection coefficients of a submerged barrier as a function of barrier height and wave length.

(2) Determination of reflection coefficients of multiple barriers.

(3) Determination of range of wave lengths effectively reduced for fixed barrier spacing.

The determination of the forces exerted on the structures is not included in this investigation.

This program is still in operation at the laboratory; this progress report presents preliminary data which will be augmented in the Final Report on the Mobile Breakwater Investigation, which is now in preparation.

II. THEORY OF WAVE REFLECTION AND INTERFERENCE

A. Wave Reflection

The nature of wave reflection may be conveniently stated in terms of the equations of the water surface as a function of two variables, time and distance. Thus, a wave traveling in the x-positive direction with amplitude a (wave height = 2a), length L, and period T, may be represented by the equation:

$$n_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{L}\right) \qquad (1)$$

If this wave is partially reflected at a point x = 0, a new wave of amplitude b and the same period and length as the original is produced, but travels in the x-negative direction:

$$n_2 = b \sin 2\pi \left(\frac{t}{T} + \frac{x}{L}\right) \qquad (2)$$

The resulting water surface in the x-negative region is given by the summation of the incident and reflected waves:

$$\begin{aligned} n &= n_1 + n_2 \\ &= (a-b) \sin 2\pi\left(\frac{t}{T} - \frac{x}{L}\right) + 2b \sin 2\pi \frac{t}{T} \cos 2\pi \frac{x}{L} \qquad (3) \end{aligned}$$

which is the equation of a standing wave of amplitude 2b, with antinode at the barrier (x = 0), superimposed on a progressive wave of amplitude (a - b) traveling in the x-positive direction. For the case of total reflection, a = b, Eq. (3) reduces to the familiar result for this situation; a standing wave of amplitude twice that of the original incident wave.

In the absence of any incidental energy losses, the amplitude of the transmitted wave is given by the law of conservation of energy, and since wave energy is proportional to the square of wave amplitude, the equation of the transmitted wave is:

$$n_3 = \sqrt{a^2 - b^2} \sin 2\pi \left(\frac{t}{T} - \frac{x}{L}\right) \tag{4}$$

The coefficient of transmission is defined as the ratio of the transmitted to incident wave amplitude, or for the case of no energy loss:

$$\rho = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \frac{b^2}{a^2}} \tag{5}$$

B. Wave Interference

Wave interference is the process of interaction of two or more wave trains. For the special case in which we are interested, the two wave trains traveling in the same direction with identical wave length, period, and amplitude, and with phase difference ϕ, may be represented by:

$$n_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{L}\right) \tag{6a}$$

$$n_2 = a \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{L}\right) + \phi\right] \tag{6b}$$

and the resultant becomes:

$$n = n_1 + n_2 = a\sqrt{2\,(1+\cos\phi)}\,\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{L}\right) + \alpha\right]$$

$$\alpha = \sin^{-1}\sqrt{\frac{1-\cos\phi}{2}} \tag{7}$$

Thus the effect of interference is to produce a new wave of the same length and period, but whose amplitude and phase are a function of the original phase difference, ϕ. The ratio of the amplitude of the resultant and original waves are plotted as a function of ϕ in Fig. 1, where it is seen that for $\phi = 0$, the amplitude is doubled, or maximum reinforcement occurs, and for $\phi = 180^\circ$, the amplitude is zero, or total cancellation occurs.

The application of the principles of wave interference is of great importance in the study of the passage of wave motion through so-called periodic or lattice structures, as for example in the theory of the transmission of electric waves through crystals or filter networks, or in the present case, the transmission of surface waves through a series of submerged breakwaters.

Each breakwater in such a series is the source of a reflected wave train traveling in the x-negative direction, and if the breakwater spacing and wave length are such that these waves reinforce, ($\phi = 0$), the amplitude of the net reflected wave due to N breakwaters will be approximately N times the amplitude for one breakwater, at least if the number N of breakwaters in the series is not too large and the individual reflection amplitudes are a small percentage of the incident wave amplitude, as seems to be the case. The energy of the net reflected wave train, however, is proportional to the square of its amplitude, hence the total energy abstracted from the incident wave train (thus not present in the transmitted wave train) is approximately N^2 times the energy removed from the incident wave train by one breakwater. The result is a rapid increase in reflective ability for a moderate number of barriers, and the possibility of utilizing a device in which each unit by itself is a very weak reflective element, but acting together achieves appreciable net energy reflection.

An important limitation of this technique is that as the number of elements are increased, the system becomes increasingly sensitive to the requirement that the interfering wave components be exactly in phase. For water waves, the proper phasing is accomplished by spacing the barriers at multiples of one-half wave length. Therefore, for an installation with a fixed spacing between elements, the range of wave lengths for which appreciable reduction of transmitted wave height is obtained narrows as the number of barriers is increased.

III. EXPERIMENTAL RESULTS

A. Application of the Theory to the Measuring Problem

The experimental work to date has been hampered by the difficulty of measuring imposed wave heights in the presence of the standing wave produced by the reflecting barriers, and by the necessity of determining the amount of energy lost at each barrier due to turbulence, and hence not appearing in the transmitted or reflected waves. If such energy losses are minor the problem is more straight forward, since Eq. (3), which represents the water surface between the wave machine and the barriers may be written in the form:

$$n = (a+b) \cos 2\pi \frac{x}{L} \sin 2\pi \frac{t}{T} - (a-b) \sin 2\pi \frac{x}{L} \cos 2\pi \frac{t}{T} \tag{8}$$

The absolute magnitude of (8), considered as a function of t is:

$$(n) = \sqrt{a^2+b^2+2ab \left(\cos^2 2\pi \frac{x}{L} - \sin^2 2\pi \frac{x}{L}\right)} \tag{9}$$

Since $\cos^2 2\pi \left(\frac{x+\frac{L}{4}}{L}\right) - \sin^2 2\pi \left(\frac{x+\frac{L}{4}}{L}\right) = -\cos^2 2\pi \frac{x}{L} + \sin^2 2\pi \frac{x}{L}$,

the sum of the squares of wave amplitude measurements made a quarter-wave length apart is:

$$n_1^2 + n_2^2 = 2\,(a^2 + b^2)$$

and since we assume the amplitude of the transmitted wave to be:

$$n_3 = \sqrt{a^2 - b^2}$$

it is possible to solve for a, the amplitude of the imposed wave, from three measurements of wave height, n_1 and n_2, a quarter-wave length apart in the region ahead of the barriers, and n_3, the transmitted wave height in the protected region behind the barriers. The coefficient of transmission would then be given by:

$$\rho = \frac{\sqrt{a^2-b^2}}{a} = \frac{n_3}{\sqrt{\frac{1}{2}\left[n_3^2 + \frac{1}{2}\left(n_1^2 + n_2^2\right)\right]}} \qquad (10)$$

If, however, a percentage N of the incident wave energy is lost by turbulence, n_3 is given by $\sqrt{(1-N)a^2-b^2}$, and with this additional variable the problem cannot be solved in this manner. For this case, it is again convenient to consider the form of the equation of the water surface given by Eq. (8):

$$n = (a+b)\cos 2\pi\frac{x}{L}\sin 2\pi\frac{t}{T} - (a-b)\sin 2\pi\frac{x}{L}\cos 2\pi\frac{t}{T}$$

It is seen that corresponding to positions where $\cos 2\pi\frac{x}{L} = 1$, the equation of the water surface is:

$$n_1 = (a+b)\sin 2\pi\frac{t}{T} \qquad (11)$$

and corresponding to position a quarter-wave length away, where $\cos 2\pi\frac{x}{L} = 0$, $\sin 2\pi\frac{x}{L} = 1$, the equation is:

$$n_2 = (a-b)\cos 2\pi\frac{t}{T} \qquad (12)$$

Thus by taking a particular case of the previously described measuring technique, where the measuring elements are not only a quarter-wave length apart, but are positioned at points of maximum and minimum vertical water motion we may solve for a and b directly

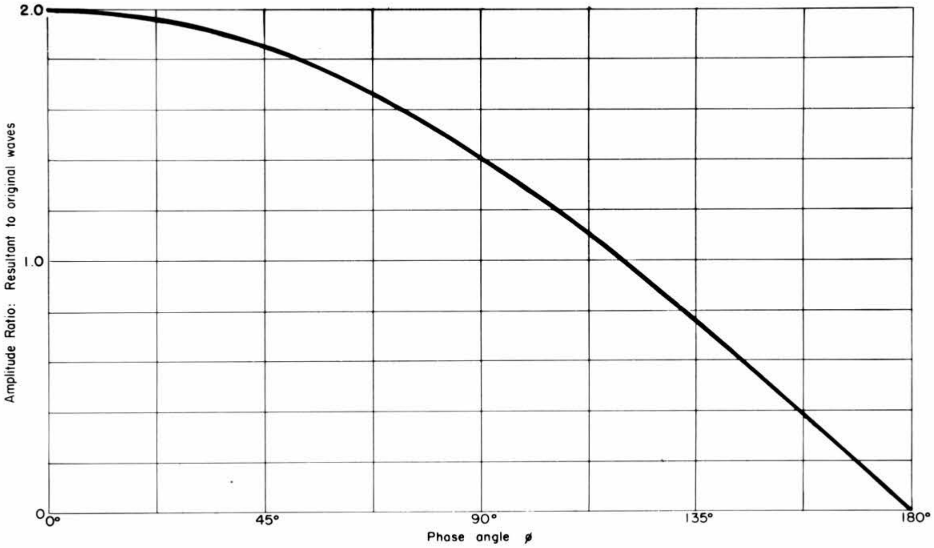

Fig. 1—Effect of interference of two identical wave trains with phase difference ø

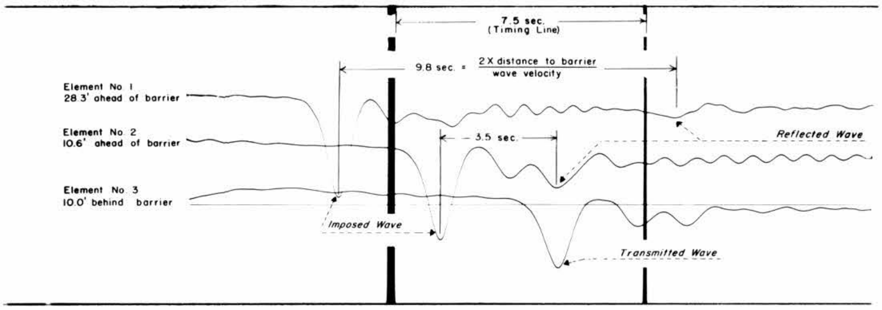

Fig. 2—Oscillograph record of partial reflection of solitary wave

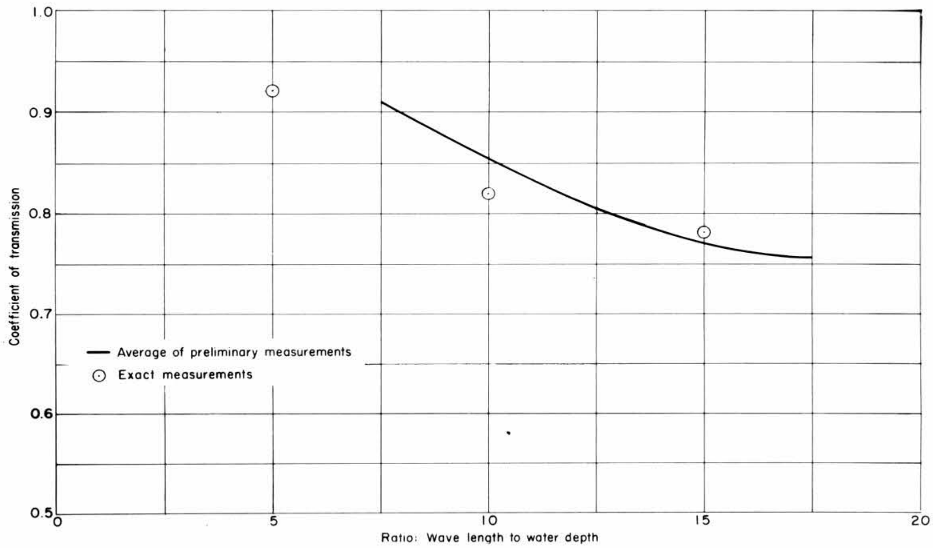

Fig. 3—Effect of wave length on barrier performance for barrier height of one-half the water depth

$$a = \frac{1}{2}(n_1 + n_2)$$
$$b = \frac{1}{2}(n_1 - n_2) \qquad (13)$$

Then, since n_3, the amplitude of the transmitted wave, is equal to $\sqrt{(1-N)\,a^2 - b^2}$

$$N = 1 - \frac{4n_3^{\,2} + (n_1 - n_2)^2}{(n_1 + n_2)^2}$$

The determination of the points of maximum and minimum motion must be a "cut-and-try" proposition, and will introduce some inaccuracy in the results.

B. Experimental Results

Solitary Wave Measurements:

A convenient method for investigating the effect of a single submerged barrier on shallow-water waves is by the use of a solitary wave, which is the archtype of shallow-water waves. The advantage of this technique is that the incident, reflected, and transmitted waves may be measured directly, thus permitting an accurate determination of the effect of barrier height for a type of wave that approximates the rather large wave length-to-depth conditions typical of harbor locations. However, the water particle motion associated with a solitary wave is decidedly different from the motion due to a train of waves, since for the solitary wave the particles do not move in closed orbits, but continually advance, hence the quantitative results of the solitary wave measurements are not strictly applicable to the case of long wave trains. The qualitative effects of relative barrier height on reflection and turbulent losses as observed with solitary waves should, however, be indicative of the trends to be expected with shallow-water wave trains.

The technique employed consisted of generating solitary waves by special manipulation of the laboratory pneumatic wave machine at one end of the 130-foot wave channel and recording the vertical motion of the water surface at points ahead of, and behind, the barrier by means of the electrical conductivity wave height measuring elements described in previous reports. The record from elements ahead of the barrier show the sequence of incident and reflected waves separated by a time interval equal to twice the distance from measuring point to barrier divided by the wave velocity. Fig. 2 is a reproduction of a typical oscillograph record. The barriers used were thin vertical walls made of 16-ga. sheet metal, reinforced at the edges and bottom.

The results of these measurements are shown in Table I. The turbulent energy loss is here calculated on the basis of the solitary wave theory of BOUSSINESQ(1), in which the wave energy is proportional to the three-halves power of the wave height, instead of the conventional AIRY(2) theory for wave trains in which the energy is proportional to the square of the wave height.

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(1) Munk, Walter H., THE SOLITARY WAVE THEORY AND ITS APPLICATION TO SURF PROBLEMS. Annals of the New York Academy of Sciences, Vol.51, Art.3. May 1949, pp.376-424.

(2) Lamb, Sir Horace, HYDRODYNAMICS DOVER PUBLICATION, New York 1945 Chapter 9, Art. 230.

TABLE I

Effect of Barrier Height on Transmission of Solitary Waves

Ratio of Barrier Height to Water Depth	Ratio of Transmitted Wave Height to Imposed Wave Height	Ratio of Reflected Wave Height to Imposed Wave Height	Per Cent of Imposed Wave Energy	
			in Reflected Wave	Lost by Turbulence
.25	.95	.00	0.0	7.4
.50	.90	.08	2.3	12.3
.67	.88	.12	4.2	13.5
.83	.75	.26	13.0	22.0

The turbulent energy loss is probably a function of the shape of the reflecting barrier, and these values for very thin walls may be larger than would be obtained for prototype structures with relatively thicker sections. The principal conclusion from these measurements is that the reflected wave accounts for very little of the energy reduction of the transmitted wave, even for large values of relative barrier height. Thus a single barrier appears to be more of a wave energy dissipating than wave reflecting device.

Preliminary Measurements with Wave Trains:

Some results are available to indicate the effect of a single barrier on wave trains. These results were obtained with a barrier height of half the water depth, and with but a single measuring element ahead of the barrier. This element therefore measured the incident wave amplitude with a possible error of plus or minus the reflected wave amplitude, $n = a \pm b$. With this limited measuring technique it is not possible to determine the division of energy reduction between the reflected wave and the turbulent losses.

Fig. 3 is a plot of the faired data for these tests, showing the theoretically expected and desirable decrease in transmission coefficient with increasing wave length-to-depth ratios; these data are accurate as a measurement of the overall operation of a barrier, but do not show the mechanism of operation as do the solitary wave measurements.

Exact Measurements:

A limited number of measurements have been completed to date employing the technique of measuring at the points of maximum and minimum water motion ahead of the barrier. These results are shown in Table II and are spotted on the plot of Fig. 3, where the agreement for coefficient of transmission is seen to be good.

TABLE II

Effect of Relative Wave Length on Transmission
Over a Single Barrier of Height Equal to One-Half the Water Depth

Ratio of Wave Length to Water Depth	Ratio of Transmitted Wave Height to Imposed Wave Height	Ratio of Reflected Wave Height to Imposed Wave Height	Per Cent of Imposed Wave Energy	
			in Reflected Wave	Lost by Turbulence
5	.92	.13	1.7	13.7
10	.82	.04	0.2	32.5
15	.78	.09	0.8	38.3

The most significant value of these measurements is that they indicate the relative magnitude of energy reflection and energy dissipation at the barrier. Although the amount of energy reflected is small, it is believed that subsequent measurements for multiple barriers will show that this mode of reduction of transmitted energy can be made significant, at least for a narrow band of wave lengths with a fixed barrier spacing.

The scatter in the values of the reflected wave height is due to the fact that this quantity is determined by the difference of two large quantities (see Eq.13), and this unfortunately necessary technique is notorious for large probable errors. It is not difficult, however, to draw the conclusion that the reflected wave contains approximately one per cent of the imposed wave energy.

IV. CONCLUSIONS

From theoretical considerations, the following conclusions are reached:

1. Submerged breakwaters offer possibilities as a mobile breakwater. It may be possible to design such structures so that they are subjected to but moderate forces while appreciably reducing imposed wave height.

2. Transmitted wave energy may be reduced by both wave reflection and energy dissipation. If several barriers are arranged in series, the amount of wave reflection may be greatly increased for certain values of imposed wave length.

The experimental program completed to date has produced the following conclusions:

1. For a single barrier, the effect of energy dissipation is much more important than wave energy reflection in reducing transmitted wave heights.

2. The coefficient of transmission of a single submerged barrier decreases with increasing wave length-to-depth ratios, a desirable situation.

3. For typical harbor conditions, such as $\frac{L}{d} = 10$, the coefficient of reflection of a single submerged breakwater with a probable maximum practical height-to-depth ratio of $\frac{1}{2}$ is approximately 0.85, corresponding to an energy reduction of nearly 30 per cent.